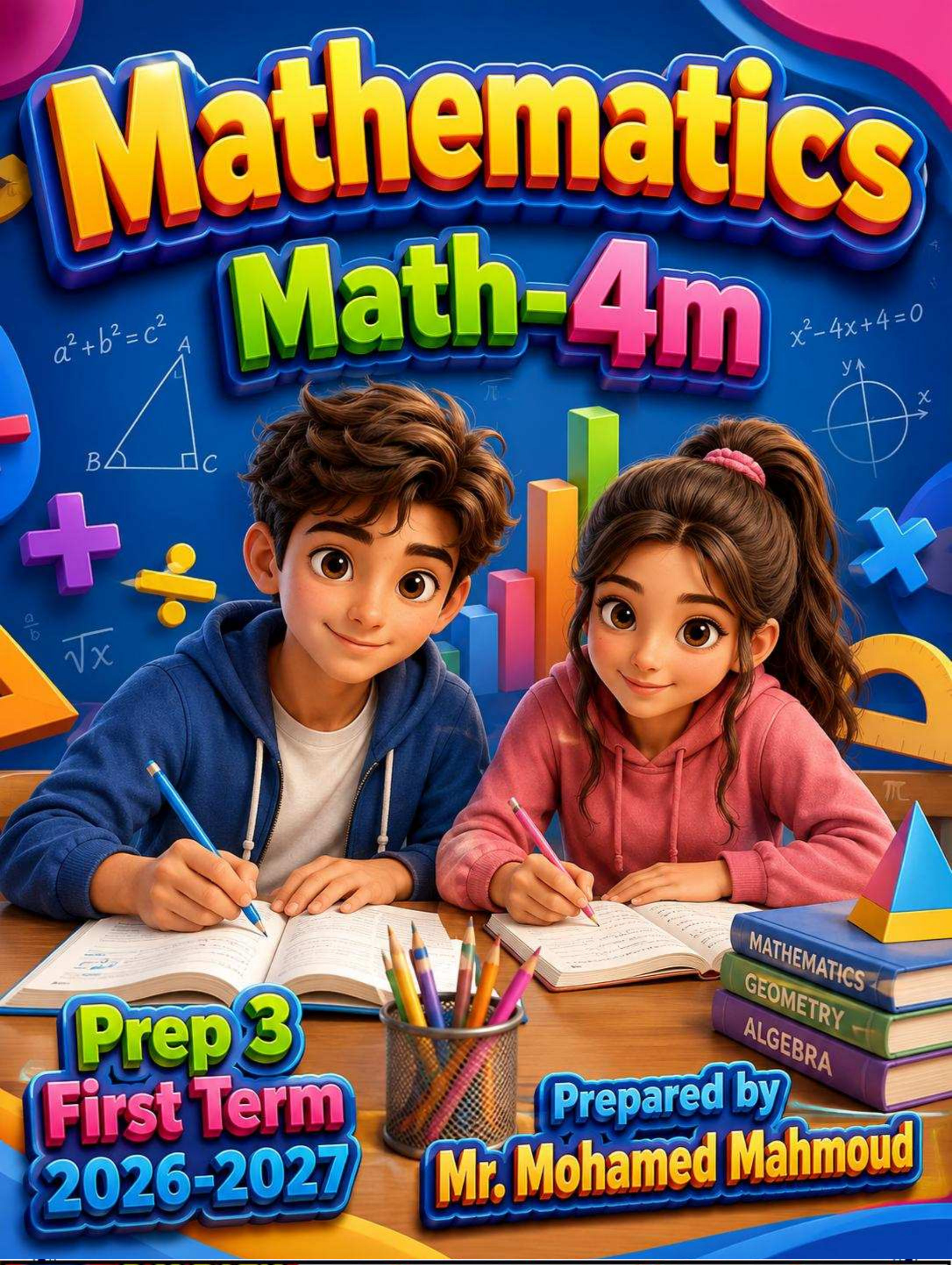
Mathematics

Math-4m



Prep 3
First Term
2026-2027

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Power of Power

$(a^m)^n$
a to power m
all to power n

Let's Learn

By the end of this lesson, you will be able to:

- Understand the meaning of power of power.
- Use the rule $(a^m)^n = a^{m \times n}$.
- Simplify expressions using power of power.
- Solve problems using power of power in different forms.
- Apply the rule in real life situations.

In Real Life

When something is repeated many times, we use powers. Sometimes, a power itself is raised to another power. This is called **"Power of Power"**.

The Basic Law

For any non-zero number a, and any integers m, n:

$$(a^m)^n = a^{m \times n}$$

Where:

- a → the base (real number, and $a \neq 0$)
- m → the power of the inner power
- n → the power of the outer power
- a^m → the inner power
- $(a^m)^n$ → the power of the power
- $a^{m \times n}$ → the simplified result

Example of symbols

$(5^3)^4$

- 4 → n (outer power)
- 3 → m (inner power)
- 5 → a (base)

Explanation

When we have a power inside another power, we multiply the two powers and keep the same base.

$(a^m)^n \longrightarrow a^{m \times n}$

$m \times n$: Multiply the exponents

In other words:

a is multiplied by itself m times, then the result is multiplied by itself n times.

This is the same as multiplying a by itself $(m \times n)$ times.

Examples

Example 1

Simplify $(2^3)^4$

Solution:

$(2^3)^4 = 2^{3 \times 4}$

$= 2^{12}$

$= 4096$

Example 2

Simplify $(3^2)^5$

Solution:

$(3^2)^5 = 3^{2 \times 5}$

$= 3^{10}$

$= 59049$

Watch Out!

Common Mistakes

- Adding the powers instead of multiplying.
- Changing the base.
- Forgetting the parentheses.

Always keep the base the same and multiply the exponents.

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More Examples

Let's see how the rule works with more examples.

1 Simplify $(2^3)^2$

Solution:

$(2^3)^2 = 2^{3\times 2}$

$= 2^6$

$= 64$

2 Simplify $(5^2)^3$

Solution:

$(5^2)^3 = 5^{2\times 3}$

$= 5^6$

$= 15625$

3 Simplify $(10^4)^2$

Solution:

$(10^4)^2 = 10^{4\times 2}$

$= 10^8$

$= 100000000$

4 Simplify $(7^3)^2$

Solution:

$(7^3)^2 = 7^{3\times 2}$

$= 7^6$

$= 117649$

5 Simplify $(3^2)^4$

Solution:

$(3^2)^4 = 3^{2\times 4}$

$= 3^8$

$= 6561$

6 Simplify $(a^5)^3$

Solution:

$(a^5)^3 = a^{5\times 3}$

$= a^{15}$

7 Simplify $(x^4)^3$

Solution:

$(x^4)^3 = x^{4\times 3}$

$= x^{12}$

8 Simplify $(y^3)^5$

Solution:

$(y^3)^5 = y^{3\times 5}$

$= y^{15}$

Key Note

When you have a power of a power:

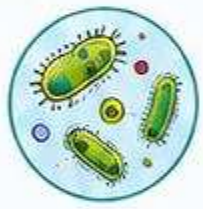
$$(a^m)^n \downarrow a^{m\times n}$$

Multiply the exponents and keep the same base.

Real Life Application

Powers of power are used in many real situations.

1 A bacteria multiplies by 10 every hour. It starts with 1 bacterium, how many will there be after 3 hours?



Solution:

Number after 1 hour $= 10^1$

Number after 2 hours $= (10^1)^2 = 10^{1\times 2} = 10^2 = 100$

Number after 3 hours $= (10^1)^3 = 10^{1\times 3} = 10^3 = 1000$

Answer: There will be 1000 bacteria.

2 A computer file increases in size by a factor of 2 every day. If it is 1 MB today, what will be its size after 4 days?

Solution:

Size after 1 day $= 2^1$

Size after 4 days $= (2^1)^4 = 2^{1\times 4} = 2^4 = 16$

Answer: The size after 4 days will be 16 MB.

You Try

Apply the rule and simplify the following.

1 $(2^2)^3$ _____

2 $(4^3)^2$ _____

3 $(5^4)^2$ _____

4 $(6^2)^4$ _____

5 $(3^5)^3$ _____

6 $(a^2)^5$ _____

7 $(m^3)^2$ _____

8 $(x^5)^2$ _____

9 $(y^4)^3$ _____

10 $(z^2)^6$ _____

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More Examples

Let's solve more examples using the rule of power of power.

1 Simplify $(2^4)^3$

Solution:

$(2^4)^3 = 2^{4\times3}$

$= 2^{12}$

$= 4096$

2 Simplify $(3^3)^4$

Solution:

$(3^3)^4 = 3^{3\times4}$

$= 3^{12}$

$= 531441$

3 Simplify $(5^2)^5$

Solution:

$(5^2)^5 = 5^{2\times5}$

$= 5^{10}$

$= 9765625$

4 Simplify $(7^2)^3$

Solution:

$(7^2)^5 = 7^{2\times3}$

$= 7^6$

$= 117649$

5 Simplify $(10^3)^2$

Solution:

$(10^3)^2 = 10^{3\times2}$

$= 10^6$

$= 1000000$

6 Simplify $(a^5)^2$

Solution:

$(a^5)^2 = a^{5\times2}$

$= a^{10}$

7 Simplify $(x^2)^4$

Solution:

$(x^2)^4 = x^{2\times4}$

$= x^8$

8 Simplify $(y^3)^3$

Solution:

$(y^3)^3 = y^{3\times3}$

$= y^9$

9 Simplify $(m^4)^3$

Solution:

$(m^4)^3 = m^{4\times3}$

$= m^{12}$

10 Simplify $(z^6)^2$

Solution:

$(z^6)^2 = z^{6\times2}$

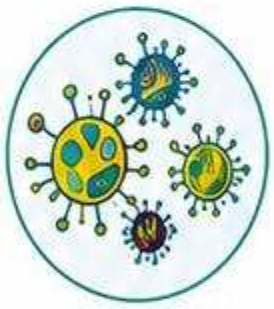
$= z^{12}$

Key Ideas

- When a power is raised to another power, multiply the exponents.
- Keep the base the same.
- Use this rule in expressions with numbers or letters.

Real Life Application

1 A virus multiplies by a factor of 10 every hour.
If there is 1 virus at the beginning,
how many will there be after 4 hours?

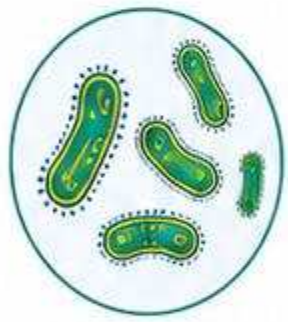


Solution:

After 1 hour $= 10^1 = 10$
After 2 hours $= 10^2 = 100$
After 3 hours $= 10^3 = 1000$
After 4 hours $= 10^4 = 10000$

Answer: There will be 10000 viruses.

2 The population of bacteria doubles every day.
If there are 500 bacteria today,
how many will there be after 5 days?



Solution:

After 1 day $= 500 \times 2^1 = 1000$
After 2 days $= 500 \times 2^2 = 2000$
After 3 days $= 500 \times 2^3 = 4000$
After 4 days $= 500 \times 2^4 = 8000$
After 5 days $= 500 \times 2^5 = 16000$

Answer: There will be 16000 bacteria.

You Try

Apply the rule and simplify the following.

1 $(2^3)^3$

2 $(4^2)^3$

3 $(5^3)^2$

4 $(3^4)^2$

5 $(6^2)^4$

6 $(a^3)^4$

7 $(x^4)^3$

8 $(y^2)^5$

9 $(m^3)^2$

10 $(z^5)^2$

Think!

If $(a^m)^n = a^{mn}$,
$a = 2$ and $m = 3$,
what is the value of n
if $2^{3n} = 2^{18}$?

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Examples (Advanced)

Let's practice more with different forms.

1 Simplify $(2^3)^2 \times (2^4)^3$

Solution:

$= 2^{3\times2} \times 2^{4\times3}$

$= 2^6 \times 2^{12}$

$= 2^{6+12}$

$= 2^{18}$

$= 262144$

2 Simplify $(3^2)^3 \div 3^4$

Solution:

$= 3^{2\times3} \div 3^4$

$= 3^6 \div 3^4$

$= 3^{6-4}$

$= 3^2$

$= 9$

3 Simplify $(5^3)^2 \times 5^4$

Solution:

$= 5^{3\times2} \times 5^4$

$= 5^6 \times 5^4$

$= 5^{6+4}$

$= 5^{10}$

$= 9765625$

4 Simplify $(7^2)^3 \div (7^3)^2$

Solution:

$= 7^{2\times3} \div 7^{3\times2}$

$= 7^6 \div 7^6$

$= 7^{6-6}$

$= 7^0$

$= 1$

5 Simplify $\dfrac{(2^4)^3 \times (2^2)^2}{2^5}$

Solution:

$= \dfrac{2^{4\times3} \times 2^{2\times2}}{2^5}$

$= \dfrac{2^{12} \times 2^4}{2^5}$

$= \dfrac{2^{16}}{2^5} = 2^{16-5}$

$= 2^{11} = 2048$

6 Simplify $\dfrac{(3^2)^4}{3^5}$

Solution:

$= \dfrac{3^{2\times4}}{3^5}$

$= \dfrac{3^8}{3^5} = 3^{8-5}$

$= 3^3 = 27$

7 Simplify $\dfrac{(x^3)^2 \times x^4}{x^7}$

Solution:

$= \dfrac{x^{3\times2} \times x^4}{x^7}$

$= \dfrac{x^6 \times x^4}{x^7}$

$= \dfrac{x^{10}}{x^7} = x^{10-7}$

$= x^3$

8 Simplify $\left(\dfrac{y^2}{y^5}\right)^3$

Solution:

$= (y^{2-5})^3$

$= (y^{-3})^3$

$= y^{-9} = \dfrac{1}{y^9}$

Key Note

Use the laws of exponents carefully:

- Multiply powers → add exponents.
- Divide powers → subtract exponents.
- Power of a power → multiply exponents.
- The base always stays the same.

Real Life Application

1 A lake is covered with 2 water lilies. Each day, the number of lilies becomes 3 times the previous day. How many lilies will cover the lake after 6 days?

Solution:

Day 1 $= 2 = 2^1$

Day 2 $= 2 \times 3 = 2 \times 3^1 = 2 \times 3^1$

Day 3 $= 2 \times 3 \times 3 = 2 \times 3^2$

Day 4 $= 2 \times 3^3$

Day 5 $= 2 \times 3^4$

Day 6 $= 2 \times 3^5 = 2 \times 243 = 486$

Answer: After 6 days, 486 lilies will cover the lake.

2 A small investment of 100 dollars earns compound interest at a rate of 5% per year, compounded annually. Find the value of the investment after 4 years.

Solution:

Year 1 $= 100(1 + 0.05)^1 = 100(1.05)^1$

Year 2 $= 100(1.05)^2$

Year 3 $= 100(1.05)^3$

Year 4 $= 100(1.05)^4 = 100(1.21550625)$

$= 121.550625$

Answer: The value after 4 years ≈ 121.55 dollars.

Practice Time

Simplify the following.

1 $(2^3)^2 \times (2^2)^3$

2 $(3^2)^3 \div 3^4$

3 $(5^2)^3 \times 5^4$

4 $(7^3)^2 \div (7^2)^3$

5 $(10^2)^3 \times (10)^4$

6 $\dfrac{(2^5)^2}{2^6}$

7 $\dfrac{(3^3)^2 \times 3^4}{3^5}$

8 $\dfrac{(x^2)^3 \times x^5}{x^4}$

9 $\left(\dfrac{a^3}{a^8}\right)^2$

10 $\left(\dfrac{m^2}{m^5}\right)^3$

Think!

If $(a^m)^n = a^{mn}$, and $2^3 = 8$, what is the value of $(2^3)^4 \div 2^2$?

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Examples (Mixed Forms)

Let's solve mixed examples that combine more than one operation.

1 Simplify $(2^3)^2 \times 2^4$

Solution:

$$(2^3)^2 \times 2^4 = 2^{3\times2} \times 2^4 = 2^6 \times 2^4 = 2^{6+4} = 2^{10} = 1024$$

2 Simplify $(3^2)^3 \div 3^4$

Solution:

$$(3^2)^3 \div 3^4 = 3^{2\times3} \div 3^4 = 3^6 \div 3^4 = 3^{6-4} = 3^2 = 9$$

3 Simplify $5^2 \times (5^3)^2$

Solution:

$$5^2 \times (5^3)^2 = 5^2 \times 5^{3\times2} = 5^2 \times 5^6 = 5^{2+6} = 5^8 = 390625$$

4 Simplify $7^5 \div (7^2)^3$

Solution:

$$7^5 \div (7^2)^2 = 7^5 \div 7^{2\times3} = 7^5 \div 7^6 = 7^{5-6} = 7^{-1} = \frac{1}{7}$$

5 Simplify $\dfrac{(2^4)^2 \times 2^3}{2^5}$

Solution:

$$\frac{(2^4)^2 \times 2^3}{2^5} = \frac{2^{4\times2} \times 2^3}{2^5} = \frac{2^8 \times 2^3}{2^5} = \frac{2^{8+3}}{2^5} = 2^{11-5} = 2^6 = 64$$

6 Simplify $\dfrac{(3^2)^3 \div 3^4}{3^2}$

Solution:

$$\frac{(3^2)^3 \div 3^4}{3^2} = \frac{3^{2\times3} \div 3^4}{3^2} = \frac{3^6 \div 3^4}{3^2} = \frac{3^{6-4}}{3^2} = \frac{3^2}{3^2} = 1$$

7 Simplify $(x^2)^3 \times x^4$

Solution:

$$(x^2)^3 \times x^4 = x^{2\times3} \times x^4 = x^6 \times x^4 = x^{6+4} = x^{10}$$

8 Simplify $\dfrac{y^7}{(y^2)^2}$

Solution:

$$\frac{y^7}{(y^2)^2} = \frac{y^7}{y^{2\times2}} = \frac{y^7}{y^4} = y^{7-4} = y^3$$

Key Ideas

- Solve inside parentheses first.
- Apply the power of a power rule.
- Multiply → add exponents.
- Divide → subtract exponents.
- Keep the base the same.

Real Life Application

1 A printer makes 2 copies in the first minute. Each minute, the number of copies is tripled. How many copies will be made in 6 minutes?

Solution:

Minute 1 = $2 = 2^1$
Minute 2 = $2 \times 3 = 2 \times 3^1 = 2^1 \times 3^1$
Minute 3 = $2 \times 3^2 = 2^1 \times 3^2$
Minute 4 = $2 \times 3^3 = 2^1 \times 3^3$
Minute 5 = $2 \times 3^4 = 2^1 \times 3^4$
Minute 6 = $2 \times 3^5 = 2 \times 243 = 486$

Answer: 486 copies will be made in 6 minutes.

2 A computer virus increases by a factor of 2^3 every hour. If it starts with 8 viruses, how many viruses will there be after 5 hours?

Solution:

After 1 hour = $8 \times (2^3)^1 = 8 \times 2^3 = 8 \times 8 = 64$
After 2 hours = $8 \times (2^3)^2 = 8 \times 2^6 = 8 \times 64 = 512$
After 3 hours = $8 \times (2^3)^3 = 8 \times 2^9 = 8 \times 512 = 4096$
After 4 hours = $8 \times (2^3)^4 = 8 \times 2^{12} = 8 \times 4096 = 32768$
After 5 hours = $8 \times (2^3)^5 = 8 \times 2^{15} = 8 \times 32768 = 262144$

Answer: There will be 262144 viruses after 5 hours.

Practice Time

Simplify the following.

1 $(2^2)^3 \times 2^2$

2 $(3^3)^2 \div 3^4$

3 $5^3 \times (5^2)^2$

4 $7^6 \div (7^3)^2$

5 $\dfrac{(2^3)^2 \times 2^4}{2^5}$

6 $\dfrac{(3^2)^3 \div 3^2}{3^3}$

7 $(x^2)^4 \times x^3$

8 $\dfrac{y^9}{(y^3)^2}$

9 $(a^3)^2 \div a$

10 $\dfrac{(z^2)^5}{z^6}$

Challenge!

If $(a^m)^n = a^{mn}$ and $(b^2)^3 \times (b^3)^2 \div b^5 = b^k$, find the value of k.

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1 Evaluate the following:

Choose the correct answer from the given choices :

(1) What is the value of 2^7 ?

(a) 64 (b) 128 (c) 256 (d) 512

(2) What is the value of $(3^6)^0$?

(a) 24 (b) 1 (c) 36 (d) 0

(3) What is the value of $\left(\frac{x^3}{x^2}\right)^2$ for $x \neq 0$?

(a) x^2 (b) x^{-2} (c) x (d) $\frac{1}{x}$

(4) What is the value of $a^8 b^4$ if $a^2 b = -2$?

(a) 16 (b) -16 (c) 8 (d) -8

(5) What is the value of $x^2 \times (x^5)^2$?

(a) x^{20} (b) x^{14} (c) x^{12} (d) x^9

(6) What is the value of 2^4 ?

(a) 15 (b) 16 (c) 32 (d) 125

(7) If $2^x = 32$, what is the value of x ?

(a) 3 (b) 4 (c) 5 (d) 6

(8) Simplify $\left(\frac{3a^2}{2b}\right)^2$.

(a) $\frac{9a^4}{4b^2}$ (b) $\frac{9a^2}{4b^2}$ (c) $\frac{6a^4}{4b^2}$ (d) $\frac{9a^4}{2b^2}$

Ali's Rule

$(a^2)^3 \times a^q = a^6 \times a^q = a^{10}$

Khaled's Rule

$(a^2)^3 \times a^q = a^6 \times a^q = a^{q+6} = a^{10}$

Discover the Mistake

Ali and Khaled solved $(a^2)^3 \times a^4$ as shown above.

Who is correct? Explain.

_____

_____

2 Apply the Concepts

A *Find each of the following in its simplest form:*

1. $(\sqrt{5})^3 \times (-\sqrt{5})^2$
2. $(a^2)^2 \times (a^3)^3$
3. $(a^4)^3 \times a^7$
4. $((-2)^3)^2 \times (-\sqrt{2})^{-4}$
5. $\frac{(a^4)^3}{(a^2)^5}$
6. $\frac{(a^4)^3}{(a^2)^5}$
8. $\frac{((-\sqrt{7})^2)^3 \times ((\sqrt{7})^3)^{-1}}{((\sqrt{7})^2)^{-2}}$

B *Express each of the following in its simplest form:*

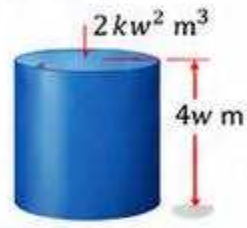
1. $(2r^2 s^{-1})^{-2}$
2. $(-2m^5 n^4)^3$
3. $(4a^4 b^3 c)^2$
4. $\frac{(c^{-1})^2 \times (-3c^2)^2}{(c^5)^2}$
5. $(-3x^2 y^3 z)^2 \times (-x^2 y^3)^2$
6. $\frac{5qp^{-2}}{(q^{-1}p)^3}$
7. $\frac{(2a^5)^3 \times (3a^3)^2}{(6a^7)^2 \times (a^{-1})^3}$
8. $\frac{(16x^2 y^{-2})^{-0}}{(4x^0 y^{-6})^{-2}}$
9. $\frac{5^0 \cdot c^2 d^3 f^{-3}}{2c^4 d^{-4} f^3}$

Real Life Skill

1. A square garden has an area of x^2 m^2. Find the length of one side of the garden.

Answer : _____

2. A cylindrical water tank has a volume of $2kw^2$ m^3 and a height of 4w m. Find the area of its base.



Answer : _____

3 More Practice

A Evaluate, then find the value of x:

1. $2^x = 8$ $x =$ _____
2. $3^4 = 81 = 9^x$ $x =$ _____
3. $5^x = 25$ $x =$ _____
4. $27^{x-1} = 9$ $x =$ _____

B Prove that:

1. $\frac{4^x \times 2^{x-4}}{16^{x-2}} = 1$
2. $\frac{(4)^x \times (49)^{x-1}}{(98)^x} = 1$
3. $\frac{27^{x-1} \times 8^x}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = 1$
4. $9^x + (27)^x$ when $3^x = 2$ = _____

4 Extension Questions

1. If $4^x = 10^2$, find the value of 2^x.

Answer : _____

2. If $a = 5^{30}$, $b = 3^{45}$, $c = 2^{60}$ arrange a, b, c in descending order.

Answer : _____

3. If $(a^m)^n = a^{mn}$ and $(b^2)^3 \times (b^3)^2 = b^5 = b^k$, find k.

$k =$ _____

4. If $16^x \div 2^{2x+1} = 8$, find x.

Answer : _____

5. If $5^x = 125$ and $x = 3^y$, find y.

Answer : _____

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1 Evaluate the following:

A *Choose the correct answer from the given choices :*

(1) What is the value of $(2^3)^4$?

(a) 2^7 (b) 2^{12} (c) 2^{14} (d) 2^{24}

(2) What is the value of $(3^2)^3 \div 3^4$?

(a) 3^2 (b) 3^3 (c) 3^4 (d) 3^6

(3) If $(5^x)^2 = 5^8$, what is the value of x ?

(a) 2 (b) 4 (c) 6 (d) 8

(4) What is the value of $a^5 \times (a^3)^2 \div a^4$?

(a) a^3 (b) a^5 (c) a^7 (d) a^9

(5) What is the value of $(10^2 \div 10^8)^3$?

(a) 10^{-18} (b) 10^{-6} (c) 10^6 (d) 10^{18}

(6) What is the value of $(x^2y^3)^3 \div (xy)^2$?

(a) x^4y^7 (b) x^4y^5 (c) x^5y^7 (d) x^5y^9

(7) If $2^x = 64$, what is the value of 2^{x-2} ?

(a) 8 (b) 16 (c) 32 (d) 64

(8) What is the value of $\frac{(2a^2)^3 \times (3a)^2}{6a^4}$?

(a) $6a^2$ (b) $9a^2$ (c) $12a^2$ (d) $18a^2$

Power of a Power Rule

If a and m, n are integers,

$(a^m)^n = a^{m \times n}$

Power of Product Rule

If a, b and n are integers,

$(ab)^n = a^n b^n$

Think Smart!

Break the expression into smaller parts and apply the rules step by step.

2 Apply the Concepts

A *Find each of the following in its simplest form:*

1 $(-3)^4 \times (-3)^2$ _____

2 $(2^2)^5$ _____

3 $(a^2)^3 \times a^7$ _____

4 $(x^3)^2 \div x^4$ _____

4 $(5^2 \times 5^3)^2$ _____

5 $(m^4 \div m^2)^3$ _____

7 $(3a^2b^3)^2$ _____

8 $(x^2y)^3 \times xy^2$ _____

B *Express each of the following in its simplest form:*

1 $(2^3 \times 4^2)^2$ _____

2 $(5a^2b)^3$ _____

3 $(3x^2y^4 \div xy^2)^2$ _____

4 $(6m^2n)^2 \div (2mn)^3$ _____

5 $(7p^5)^0$ _____

6 $(a^2b^5)^4 \div (ab^2)^3$ _____

7 $(12x^4y^3 \div 3x^2y)^2$ _____

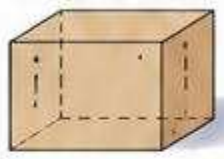
8 $(-2a^3)^2 \times (-3a^2)^3$ _____

Real Life Skill

1 A square swimming pool has an area of 25 m^2. What is the length of one side of the pool?

Answer : _____

2 A cube-shaped box has a volume of 64 cm^3. What is the length of one edge of the box?



Answer : _____

3 More Practice

A *Find the value of x:*

1 $2^{x+1} = 32$

$x =$ _____

2 $3^{2x} = 81$

$x =$ _____

3 $(5^x)^2 = 125$

$x =$ _____

4 $4^x \div 4^3 = 16$

$x =$ _____

5 $(2^x)^3 = 64$

$x =$ _____

6 $27^{x-1} = 9$

$x =$ _____

B *Simplify the following:*

1 $(2^2 \times 3^3)^2$ _____

2 $(a^3b^2)^2 \times ab^4$ _____

6 $(x^2y^3)^2 \div (xy)^3$ _____

4 $(8a^2b^3 \div 2ab)^2$ _____

5 $(3^2 \times 5^2 \times 7^2)^0$ _____

6 $(p^4q + p^2q^5)^3$ _____

4 Extension Questions

1 If $(2^x)^3 = 64$, find the value of x.

Answer : _____

2 If $a = 2^5$, $b = 3^4$ and $c = 5^2$, arrange a, b, c in descending order.

Answer : _____

3 If $(a^2b)^3 = a^m b^n$ and $(a^2b)^3 = a^m b^n$, find the value of m and n.

m = _____, n = _____

4 If $(3 \times 5)^x = 15^2$, find the value of x.

Answer : _____

5 If $2^x = 5$ and $2^y = 3$, find the value of 2^{x+y}.

Answer : _____

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1 Evaluate the following:

A *Choose the correct answer from the given choices :*

(1) What is the value of $(2^3)^4$?

(a) 2^7 (b) 2^{12} (c) 2^{14} (d) 2^{24}

(2) What is the value of $5^2 \times 5^3 \times 5$?

(a) 5^2 (b) 5^3 (c) 5^5 (d) 5^6

(3) What is the value of $(-3)^4 \times (-3)^2$?

(a) -3^2 (b) 3^6 (c) -3^6 (d) 3^8

(4) If $9^x = 27$, what is the value of x ?

(a) 2 (b) 3 (c) 4 (d) 6

(5) What is the value of $(10^3 \div 10)^2$?

(a) 10^2 (b) 10^4 (c) 10^6 (d) 10^8

(6) What is the value of $(x^2y^3)^2 \times (xy)^3$?

(a) x^4y^9 (b) x^4y^6 (c) x^5y^9 (d) x^7y^9

(7) If $2^x = 128$, what is the value of 2^{x-3} ?

(a) 8 (b) 16 (c) 32 (d) 64

(8) What is the value of $\dfrac{(3a^2b)^3 \times (2ab^2)^2}{(6a^2b^3)^2}$?

(a) $3a^2b^2$ (b) $3ab^2$ (c) $3a^4b$ (d) $3ab$

Quotient of Powers Rule

If $a \neq 0$ and m, n are integers,

$a^m \div a^n = a^{m-n}$

Power of a Power Rule

If a and m, n are integers,

$(a^m)^n = a^{m \times n}$

Remember!

When multiplying or dividing powers with the same base, keep the base and add or subtract the exponents.

2 Apply the Concepts

A *Find each of the following in its simplest form:*

1 $(-2)^5 \times (-2)^3$ _____

2 $(3^2)^4$ _____

3 $(a^3)^2 \times a^5$ _____

4 $(x^4)^3 \div x^5$ _____

5 $(4^2 \times 4^3)^2$ _____

6 $(m^3 \div m)^2$ _____

7 $(2a^2b^3)^3$ _____

8 $(x^3y^2)^2 \times xy^4$ _____

B *Express each of the following in its simplest form:*

1 $(2^4 \div 2^2)^3$ _____

2 $(5a^2)^2 \times (2a^3)^3$ _____

3 $(3x^2y^3 \div xy)^2$ _____

4 $(6m^2n)^3 \div (3mn^2)^2$ _____

5 $(7p^4q)^0$ _____

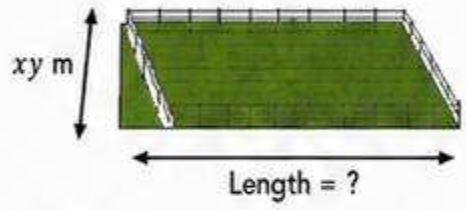
6 $(a^2b^3)^2 \div (ab^2)^4$ _____

7 $(8x^4y^2 \div 4x^2y)^2$ _____

8 $(-3a^2)^2 \times (-2a^3)^2$ _____

Real Life Skill

1 A rectangular field has an area of $x^2y^3\ m^2$. If its width is xy m, what is its length ?



Answer : _____

2 A cube-shaped ice box has a volume of $125a^2b^6$ cm^3. What is the length of one edge?

Answer : _____

3 More Practice

A *Find the value of x:*

1 $2^x = 64$
$x =$ _____

2 $3^x = 243$
$x =$ _____

3 $(5^x)^2 = 625$
$x =$ _____

4 $4^x \div 4^2 = 16$
$x =$ _____

5 $9^x \times 9 = 729$
$x =$ _____

6 $2^{x-1} = 32$
$x =$ _____

B *Simplify the following:*

1 $(2^3 \times 2^{-1})^2$ _____

2 $(a^2b^3)^3 \times (ab^2)^2$ _____

3 $(3x^2y)^2 \div (xy^2)^3$ _____

4 $(4a^2b)^2 \div 2ab$ _____

5 $(5^2 \times 5^{-3})^0$ _____

6 $(p^3q^2)^2 \times p^2q^3$ _____

4 Extension Questions

1 If $4^x = 256$, find the value of 4^{x-2}.

Answer : _____

2 If $a = 2^4$, $b = 3^2$ and $c = 2^3 \times 3$, arrange a, b, c in ascending order.

Answer : _____

3 If $(a^m)^n = a^{mn}$ and $(a^3b^2)^2 = a^mb^n$, find the value of m and n.

$m =$ _____, $n =$ _____

4 If $9^x = 81$, find the value of x.

Answer : _____

5 If $2^x = 8$ and $2^y = 3$, find the value of 2^{x+y}.

Answer : _____

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1 Concept Check

A *Fill in the blanks:*

1. $3^5 \times 3^2 = 3$ _____
2. $a^7 \div a^3 =$ _____
3. $(x^4)^3 =$ _____
4. $2^5 \times 4^5 =$ _____
5. $\left(\frac{6^2}{6^5}\right) = 6$ _____
6. $27^x = 3^9 \Rightarrow x =$ _____

B *Write each of the following in a single power of the given base:*

1. $5^6 \times 5^3 \times 5^2$ (base 5) = _____
2. $2^8 \div 2^3$ (base 2) = _____
3. $7^4 \times 7^{-2} \times 7^3$ (base 7) = _____
4. $a^9 \div a^4 \times a^2$ (base a) = _____

C *Compare using $>$, $<$ or $=$:*

1. 2^5 ◯ 2^3
2. $3^2 \times 3^4$ ◯ 3^7
3. 5^6 ◯ 25^3
4. 8^2 ◯ 4^3

D *Match the expressions in column (A) with their equivalent in column (B):*

Column (A)	Column (B)
1 $(2^3)^4$	a 4^2
2 $3^5 \div 3^{-2}$	b 2^{12}
3 $(5^{-2})^3$	c 5^{-6}
4 $4^3 \times 4^{-1}$	d 3^7

Think!

If $x^m = 2^5$ and $x^n = 2^3$, find x^{m-n}.

Explain your answer.

_____

_____

2 Problem Solving

1. A bacteria cell divides into 8 cells every hour. If there is 1 cell at the beginning, how many cells will there be after 6 hours?

Answer : _____

2. A tiny rocket travels 3^4 km in the first second. If it travels the same distance tripled each second, how far does it travel in the 3^{rd} second?

Answer : _____

3. A factory produces 2^6 items on Monday. If the production is doubled each day, how many items will be produced on Thursday?

Answer : _____

4. The side of a square is 3^5 cm. Find the area of the square.

Answer : _____

Real Life Connection

A microscope can magnify an object by a factor of 10^3 each time you increase the power by one level.

If the magnification at level 1 is 10^3 times, what is the magnification at level 4?

Answer : _____

3 Challenge Yourself

Choose the correct answer:

1. Which of the following is equal to $\left(\frac{2^4 \times 3^2}{2^2}\right)^2$?

a) $2^4 \times 3^4$ b) $2^8 \times 3^2$ c) $2^4 \times 3^2$ d) $2^6 \times 3^4$

2. Which expression is equal to $5^3 \div 5^{-4} \times 5^2$?

a) 5^{-1} b) 5^1 c) 5^9 d) 5^5

3. If $9^x = 27^2$, then x equals:

a) 2 b) 3 c) 4 d) 6

4. Which of the following is **NOT** equal to $(2a^3b^2)^2$?

a) $4a^6b^4$ b) $2^2a^6b^4$ c) $2a^6b^4$

5. Which is equal to $\frac{(x^3y^{-2})^4}{x^5y^3}$?

a) x^7y^{-5} b) x^7y^{-11} c) x^7y^{-8} d) $x^{-7}y^{-11}$

6. If $4^m = 2^{15}$, then the value of m is:

a) 5 b) 10 c) 15 d) 30

4 Mixed Practice

A *Simplify:*

1. $(3^2)^3 \times 3^{-4}$
2. $6^4 \div 6^2 \times 6^{-1}$
3. $(2^3 \times 5^2)^2$
4. $\frac{(7^3)^2 \times 7^{-4}}{7^3}$
5. $(x^2y^{-1})^3 \times (xy^2)^2$
6. $\frac{(a^4b^3)^2 \div (a^2b^{-1})^3}{(a^3b)^2}$

B *Find the value:*

1. $2^x = 64$
2. $3^{x-1} = 27$
3. $5^x \times 5^2 = 5^7$
4. $\frac{7^x}{7^3} = 49$

C *Arrange in ascending order:*

1. 2^5 , 4^2 , 8 , 2^4

_____

2. 3^3 , 9 , 27 , 3^4

_____

D *Write as a single power:*

1. $2^3 \times 2^5 \times 2 \div 2^2 =$ _____
2. $a^4 \times a^{-2} \div a^3 =$ _____

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Homework

Keep Practicing!

1 Evaluate each of the following.

1. $2^4 =$ _____
2. $3^3 =$ _____
3. $5^2 =$ _____
4. $7^1 =$ _____
5. $4^3 =$ _____
6. $2^6 =$ _____
7. $3^0 =$ _____
8. $10^2 =$ _____
9. $(2^2)^3 =$ _____
10. $(3^3)^2 =$ _____
11. $(5^2)^4 =$ _____
12. $(7^3)^1 =$ _____
13. $2^3 \times 2^4 =$ _____
14. $3^5 \div 3^2 =$ _____
15. $5^2 \times 5^3 =$ _____
16. $7^6 \div 7^2 =$ _____
17. $8^2 \times 2^3 =$ _____
18. $9^3 \div 3^2 =$ _____
19. $11^2 \times 11^3 =$ _____
20. $12^5 \div 12^2 =$ _____

2 Simplify each expression.

1. $(a^2)^3 =$ _____
2. $(x^3)^2 =$ _____
3. $(b^4)^2 =$ _____
4. $(y^5)^1 =$ _____
5. $a^3 \times a^4 =$ _____
6. $x^5 \div x^2 =$ _____
7. $b^6 \times b^3 =$ _____
8. $y^9 \div y^4 =$ _____
9. $2^3 \times 2^2 =$ _____
10. $3^7 \div 3^3 =$ _____
11. $5^4 \times 5 =$ _____
12. $7^8 \div 7^2 =$ _____
13. $(a^2)^3 \times a =$ _____
14. $(x^3)^2 \div x^4 =$ _____
15. $(m^2)^4 \times m^3 =$ _____
16. $(n^5)^2 \div n^3 =$ _____

3 Write each of the following as a single power.

1. $2^3 \times 2^5 =$ _____
2. $3^6 \div 3^2 =$ _____
3. $5^4 \times 5^3 =$ _____
4. $7^9 \div 7^4 =$ _____
5. $a^2 \times a^5 =$ _____
6. $b^7 \div b^3 =$ _____
7. $x^4 \times x^4 =$ _____
8. $y^8 \div y^2 =$ _____
9. $2^3 \times 2^2 \times 2^4 =$ _____
10. $3^6 \div 3^2 \times 3^3 =$ _____
11. $a^2 \times a^{-1} =$ _____
12. $b^7 \div b^4 \times b^2 =$ _____

4 Solve the following problems.

1. A newspaper doubles the number of copies printed each day. If 2^5 copies are printed on the first day, how many copies will be printed on the sixth day?

   Answer: _____

2. A computer virus increases by a factor of 3 every hour. If it starts with 2 viruses, how many viruses will there be after 5 hours?

   Answer: _____

3. A certain plant grows in height by a factor of 2 each week. If its height was 2^2 cm in the first week, what will be its height in the 5^{th} week?

   Answer: _____

4. A printer makes 2^4 pages in the first minute. Each minute, the number of pages it makes is multiplied by 2. How many pages will it make in 4 minutes?

   Answer: _____

5 Choose the correct answer.

1. $(2^3)^2$ is equal to:
   a) 2^5 b) 2^6 c) 2^8 d) 2^{12}
2. $3^4 \times 3^{-2}$ is equal to:
   a) 3^2 b) 3^6 c) 3^{-2} d) 3^8
3. If $5^x = 125$, then x equals:
   a) 2 b) 3 c) 4 d) 5
4. $(a^2b^3)^2$ is equal to:
   a) a^4b^6 b) a^2b^6 c) a^4b^5 d) a^3b^6
5. $2^m \times 2^n$ is equal to:
   a) 2^{m+n} b) 2^{m-n} c) $2^{m \times n}$ d) $2^{m/n}$
6. $(x^3)^4 \div x^5$ is equal to:
   a) x^7 b) x^8 c) x^{12} d) x^9

6 Think and Answer

1. If $2^a = 32$ and $2^b = 8$, find the value of $a - b$.

   _____

2. Find the smallest natural number n such that $3^n > 100$.

   _____

3. If $4^x = 64$ and $8^y = 64$, compare x and y.

   _____

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Homework

Keep Practicing!

1 Evaluate each of the following.

1. a^2 when $a = 3$
   $a^2 =$ _____
2. b^3 when $b = -2$
   $b^3 =$ _____
3. $(2x)^2$ when $x = 5$
   $(2x)^2 =$ _____
4. y^2 when $y = -4$
   $y^2 =$ _____
5. $(3m)^3$ when $m = 2$
   $(3m)^3 =$ _____
6. $(k^2)^3$ when $k = -3$
   $(k^2)^3 =$ _____
7. $a^3 - b^2$ when $a = 2$, $b = 5$
   $a^3 - b^2 =$ _____
8. $2x^2 + y^3$ when $x = 3$, $y = -1$
   $2x^2 + y^3 =$ _____
9. $3p^2q - q^3$ when $p = 2$, $q = -2$
   $3p^2q - q^3 =$ _____
10. a^2b when $a = -2$, $b = 3$
    $a^2b =$ _____
11. $2m^2n^2$ when $m = -1$, $n = 4$
    $2m^2n^2 =$ _____
12. x^3y^2 when $x = 2$, $y = -3$
    $x^3y^2 =$ _____

2 Simplify each of the following expressions.

1. $a^2 \times a^5 =$ _____
2. $b^7 \div b^3 =$ _____
3. $x^4 \times x^2 \times x =$ _____
4. $y^9 \div y^4 \times y^2 =$ _____
5. $(m^3)^2 =$ _____
6. $(a^2b)^3 =$ _____
7. $(2x^2y)^2 =$ _____
8. $(-3a^3b^2)^2 =$ _____
9. $p^6 \div p^2 \times p^3 =$ _____
10. $c^5 \times c^3 \div c^4 =$ _____
11. $(d^2)^3 \times d^4 =$ _____
12. $(e^3f)^2 \div (ef^2) =$ _____
13. $(xy^2)^3 =$ _____
14. $(x^2y^3) \div (xy)^2 =$ _____
15. $(2a^2)^3 \times (a^4)^2 =$ _____
16. $(-x^3y)^2 \times (x^2y^2) =$ _____

3 Write each of the following as a single power.

1. $a^3 \times a^4 =$ _____
2. $b^8 \div b^5 =$ _____
3. $x^6 \times x \times x^3 =$ _____
4. $y^7 \div y \times y^2 =$ _____
5. $(a^2)^4 =$ _____
6. $(b^3)^2 =$ _____
7. $(x^2y^3)^2 =$ _____
8. $(ab^2)^3 =$ _____
9. $p^4 \times p^3 \div p^2 =$ _____
10. $q^9 \div q^3 \times q =$ _____
11. $(m^2n)^3 \div (mn) =$ _____
12. $(2x^2y)^3 \div (2xy)^2 =$ _____

4 Express each of the following in its simplest form.

1. $(2a^4 \times 3a^2) \div (6a^3) =$ _____
2. $(9x^3 \times 2x) \div (3x^2) =$ _____
3. $(4b^5 \times 5b^2) \div (2b^3) =$ _____
4. $(6m^4n^2 \times 3mn) \div (9m^2n) =$ _____
5. $(8x^2y^3) \div (2xy) =$ _____
6. $(15a^3b^2) \div (3ab^2) =$ _____
7. $(12p^4q \times 5pq^2) \div (10p^2q) =$ _____
8. $(14m^2n^3) \div (7mn) =$ _____
9. $(4x^2y)^2 \div (2xy) =$ _____
10. $(3a^3b)^2 \times (2ab^2) =$ _____
11. $(2p^2q^3)^3 \div (4pq) =$ _____
12. $(-3x^2y)^2 \times (-2xy) =$ _____

5 Solve the following problems.

1. A rectangle has a length of $3x^2$ m and a width of $2x$ m.
   Find the area of the rectangle in terms of x.
   Answer : _____
2. The volume of a cube is $8a^3$ cm^3.
   Find the length of one edge of the cube in terms of a.
   Answer : _____
3. The number of bacteria in a dish is n. If the number triples every hour, write an expression for the number of bacteria after 4 hours.
   Answer : _____
4. A machine produces p^2 items in one minute. If the number of items doubles every minute, write an expression for the number of items produced in 5 minutes.
   Answer : _____

6 Challenge Yourself

1. Which of the following is equal to $(a^2b^3)^2 \div (ab)$?
   a) ab^5 b) a^3b^5 c) a^3b^6 d) a^5b^5
2. If $x^m \times x^3 = x^7$, what is the value of m ?
   a) 1 b) 3 c) 4 d) 7
3. Which expression is NOT equal to x^6y^3 ?
   a) $(x^2)^3\, y^3$ b) $x^3 \times x^2y^3$
   c) $(xy)^3\, x^3$ d) $x^4y^3 \times x^2$
4. If $2^a = 32$ and $2^b = 8$, what is the value of $a - b$?
   a) 1 b) 2 c) 3 d) 4
5. Which of the following is equal to $\dfrac{(x^3y^2)^2 \times (xy)^3}{x^4y^3}$?
   a) x^2y^2 b) x^3y^2
   c) x^2y^3 d) x^3y^3
6. If $(-2m^2n)^3 = -8m^6n^3$, what is the value of m^6n^3 ?
   a) 1 b) 2 c) -1 d) -2

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Exponential Equations

Bacteria under a microscope

Get Inspired!

In nature, bacteria multiply rapidly. Some types can double every hour under ideal conditions using binary fission.

This rapid growth can be modeled using exponential equations.

If a single bacterium grows in this way, how many bacteria will be present after several hours?

It may reach 1024 bacteria!

In this lesson, we will learn what exponential equations are and how to solve them using the laws of exponents.

Learning Outcomes

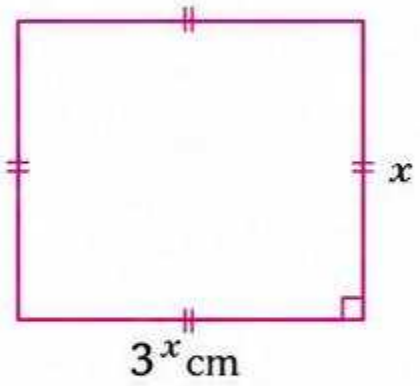
- Solve exponential equations.
- Use the laws of exponents.
- Solve real-life problems using exponential equations.

Key Vocabulary

- Exponential equation
- Base
- Exponent
- Equality
- Solution of an equation

Think!

If the area of the square opposite equals 63 cm^2, what is the value of x?



Learn

Basic Rule

An exponential equation is an equation that contains a variable in the exponent.

Examples of exponential equations:

- $x^2 = 9$
- $x - 7 = 3^2$

Examples of non-exponential equations:

- $2^x = 8$
- $3^{x-4} = \frac{1}{9}$

Solving an exponential equation means finding the value(s) of the variable that make the two sides of the equation equal.

Solving exponential equations

Solve the following exponential equation using the equality property.

If $a = b$, then:

1) If $a^m = a^n$, and $a > 0$, $a \neq 1$, then $m = n$

2) If $a^m = b^m$, and $a > 0$, $a \neq 1$, then $a = b$

Examples:

- If $3^x = 3^4$, and $3 > 0$, $3 \neq 1$, then $x = 4$
- If $5^x = 5^7$, and $5 > 0$, $5 \neq 1$, then $x = 7$

Remember

You can use repeated multiplication to write numbers in exponential form.

125	5
25	5
5	5
1	

$125 = 5^3$

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Examples

Solve each of the following exponential equations using the equality property.

1 $2^x = 16$

① $2^x = 16$

② $2^x = 2^4$ $(16 = 2^4)$

③ $x = 4$

$\therefore$ $\boxed{x = 4}$

2 $3^x = 81$

① $3^x = 81$

② $3^x = 3^4$ $(81 = 3^4)$

③ $x = 4$

$\therefore$ $\boxed{x = 4}$

3 $5^x = 125$

① $5^x = 125$

② $5^x = 5^3$ $(125 = 5^3)$

③ $x = 3$

$\therefore$ $\boxed{x = 3}$

4 $7^x = 49$

① $7^x = 49$

② $7^x = 7^2$ $(49 = 7^2)$

③ $x = 2$

$\therefore$ $\boxed{x = 2}$

5 $10^x = 1000$

① $10^x = 1000$

② $10^x = 10^3$ $(1000 = 10^3)$

③ $x = 3$

$\therefore$ $\boxed{x = 3}$

6 $2^{x-1} = 8$

① $2^{x-1} = 8$

② $2^{x-1} = 2^3$ $(8 = 2^3)$

③ $x - 1 = 3$

④ $x = 4$

$\therefore$ $\boxed{x = 4}$

7 $3^{x+2} = 27$

① $3^{x+2} = 27$

② $3^{x+2} = 3^3$ $(27 = 3^3)$

③ $x + 2 = 3$

④ $x = 1$

$\therefore$ $\boxed{x = 1}$

8 $5^{2x} = 125$

① $5^{2x} = 125$

② $5^{2x} = 5^3$ $(125 = 5^3)$

③ $2x = 3$

④ $x = \frac{3}{2}$

$\therefore$ $\boxed{x = \frac{3}{2}}$

9 $4^{x-2} = 16$

① $4^{x-2} = 16$

② $4^{x-2} = 4^2$ $(16 = 4^2)$

③ $x - 2 = 2$

④ $x = 4$

$\therefore$ $\boxed{x = 4}$

Practice Yourself

Solve each equation using the equality property.

1) $2^x = 64$ 2) $3^x = 243$ 3) $5^x = 1$ 4) $7^x = 1$ 5) $10^x = 10{,}000$

6) $2^{x-1} = 4$ 7) $3^{x+1} = 81$ 8) $5^{2x} = 25$ 9) $4^{x-1} = 2$ 10) $9^{x+1} = 27$

11) $2^{x+3} = 16$ 12) $5^{x-2} = 25$ 13) $3^{2x} = 27$ 14) $7^{x-1} = 7^2$ 15) $10^{x+2} = 100$

Key Idea

If the bases of the two sides are the same, we can equate the exponents.

$$a^m = a^n \ , \ a > 0, \ a \neq 1$$
$$\Rightarrow \ m = n$$

Common Mistakes

✗ Adding the exponents instead of equating them.

Example:

$3^x = 3^2$

Wrong: $x = 3 + 2 = 5$

Correct: $x = 2$

✗ Ignoring the base of each side.

Example:

$2^x = 3^2$

Wrong: $x = 2$

Correct: Not possible because the bases are different.

✗ Forgetting the order of operations with exponents containing expressions.

Example:

$2^{x+1} = 8$

Wrong: $x + 1 = 8$

Correct: $2^{x+1} = 2^3 \Rightarrow x + 1 = 3$

Remember

- ✓ The bases must be the same.
- ✓ The bases must be positive.
- ✓ The bases must not equal 1.
- ✓ Then, equate the exponents.

Quick Check

Fill in the blank:

1) If $2^x = 32$, then $x =$ _____.
2) If $5^x = 625$, then $x =$ _____.
3) If $10^{x-1} = 100$, then $x =$ _____.

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More Examples

Solve each of the following exponential equations.

1 $2^{x+3} = 16$

① $2^{x+3} = 16$

② $2^{x+3} = 2^4$ $(16 = 2^4)$

③ $x + 3 = 4$

④ $x = 4 - 3$

$\therefore \boxed{x = 1}$

2 $3^{x-1} = 81$

① $3^{x-1} = 81$

② $3^{x-1} = 3^4$ $(81 = 3^4)$

③ $x - 1 = 4$

④ $x = 4 + 1$

$\therefore \boxed{x = 5}$

3 $5^{2x} = 625$

① $5^{2x} = 625$

② $5^{2x} = 5^4$ $(625 = 5^4)$

③ $2x = 4$

④ $x = \frac{4}{2}$

$\therefore \boxed{x = 2}$

4 $7^{x-2} = 49$

① $7^{x-2} = 49$

② $7^{x-2} = 7^2$ $(49 = 7^2)$

③ $x - 2 = 2$

④ $x = 2 + 2$

$\therefore \boxed{x = 4}$

5 $4^{x+1} = 64$

① $4^{x+1} = 64$

② $4^{x+1} = 4^3$ $(64 = 4^3)$

③ $x + 1 = 3$

④ $x = 3 - 1$

$\therefore \boxed{x = 2}$

6 $8^{x-1} = \frac{1}{8}$

① $8^{x-1} = \frac{1}{8}$

② $8^{x-1} = 8^{-1}$ $(\frac{1}{8} = 8^{-1})$

③ $x - 1 = -1$

④ $x = -1 + 1$

$\therefore \boxed{x = 0}$

7 $9^{x+2} = 729$

① $9^{x+2} = 729$

② $9^{x+2} = 9^3$ $(729 = 9^3)$

③ $x + 2 = 3$

④ $x = 3 - 2$

$\therefore \boxed{x = 1}$

8 $2^{2x-1} = 32$

① $2^{2x-1} = 32$

② $2^{2x-1} = 2^5$ $(32 = 2^5)$

③ $2x - 1 = 5$

④ $2x = 6$

⑤ $x = 3$

$\therefore \boxed{x = 3}$

Special Cases

1) $a^0 = 1$ (where $a \neq 0$)

- Examples:

$3^x = 1 \Rightarrow 3^x = 3^0 \Rightarrow x = 0$

$7^{x+2} = 1 \Rightarrow 7^{x+2} = 7^0 \Rightarrow x + 2 = 0 \Rightarrow x = -2$

2) $a^1 = a$

- Examples:

$5^x = 5 \Rightarrow 5^x = 5^1 \Rightarrow x = 1$

$2^{x-3} = 2 \Rightarrow 2^{x-3} = 2^1 \Rightarrow x - 3 = 1 \Rightarrow x = 4$

Practice Yourself

Solve each equation using the equality property.

1 $2^{x+2} = 64$ _____

2 $3^{x-2} = 27$ _____

3 $5^{x+1} = 125$ _____

4 $7^{x+1} = 343$ _____

5 $4^{2x} = 256$ _____

6 $9^{x-1} = 81$ _____

7 $2^{3x} = 64$ _____

8 $5^{x-2} = 5$ _____

9 $3^x = 1$ _____

10 $7^{x+3} = 1$ _____

11 $2^{x-3} = 8$ _____

12 $9^{x+2} = 81$ _____

13 $4^{x-1} = 16$ _____

14 $10^{x-1} = 100$ _____

15 $8^{x+1} = 2$ _____

Key Idea

Make the bases the same using the laws of exponents, then equate the exponents.

Laws of Exponents (Useful)

1) $a^m \cdot a^n = a^{m+n}$

2) $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$

3) $(a^m)^n = a^{mn}$

4) $a^0 = 1$, $a \neq 0$

5) $a^{-m} = \frac{1}{a^m}$, $a \neq 0$

Common Mistakes

1) Forgetting to write both sides with the same base.

Example:

$4^{x+1} = 16$

Wrong: $4^{x+1} = 16 \Rightarrow x + 1 = 16$

Correct: $4^{x+1} = 2^4 = 4^2 \Rightarrow x + 1 = 2$

2) Ignoring negative exponents.

Example:

$2^x = \frac{1}{8}$

Wrong: $2^x = 8^{-1} = 8 \Rightarrow x = 3$

Correct: $2^x = 2^{-3} \Rightarrow x = -3$

3) Mistake in solving the linear equation after equating exponents.

Example:

$3^{x-2} = 27$

Wrong: $x - 2 = 3 \Rightarrow x = 1$

Correct: $x - 2 = 3 \Rightarrow x = 5$

Think!

If $2^{x+1} = 32$,

what is the value of x?

_____

Quick Check

Fill in the blank:

1) If $3^x = 27$, then $x =$ _____.

2) If $5^{x+1} = 125$, then $x =$ _____.

3) If $2^{x-1} = 1$, then $x =$ _____.

4) If $4^{2x} = 16$, then $x =$ _____.

5) If $7^{x+2} = 49$, then $x =$ _____.

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2 $2^x = 16^{3-x}$

$\therefore\ 2^x = (2^4)^{3-x}$

$\therefore\ 2^x = 2^{4(3-x)}$

$\therefore\ 2^x = 2^{12-4x}$

$\therefore\ 2^x = 12 - 4x$

$\therefore\ x^2 + 4x - 12 = 0$

$\therefore\ (x+6)(x-2) = 0$

$\therefore x + 6 = 0$ *or* $x - 2 = 0$

$x = -6$ | $x = 2$

4 $: \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{4}{9}\right)^{2x-7}$

$\therefore\ \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{2^2}{3^2}\right)^{2x-7}$

$\therefore\ \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{2}{3}\right)^{2(2x-7)}$

$\therefore\ \left(\frac{2}{3}\right)^{6-3x} = \left(\frac{2}{3}\right)^{4x-14}$

$\therefore\ 6 - 3x = 4x - 14$

$\therefore\ 5 - 3x = -4x + 14$

$\therefore\ 4x - 3x = 14 - 5$

$\therefore\ x = 9$

3 $\left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{4}\right)^{2x-3}$

$\therefore\ \left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{2^2}\right)^{2x-3}$

$\therefore\ \left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{2}\right)^{2(2x-3)}$

$\therefore\ \left(\frac{1}{2}\right)^{5-x} = \left(\frac{1}{2}\right)^{4x-6}$

$\therefore\ 5 - x = 4x - 6$

$\therefore\ 5 + 6 = 4x + x$

$\therefore\ 11 = 5x$

$\therefore\ x = \frac{11}{5}$

Note :

$x = 0$: if $a \neq 1,\ a > 0$ then $a^x = 1$ • For example : $5^0 = 1$

$x = 0$: if $a \neq b,\ b > 0,\ a > 0$ then $a^x = b^x$ • For example : $3^0 = 7^0$

Example 3

Solve both equations with unequal bases in $\mathbb{R}$.

1 $2^{x+1} = 3^{x+1}$

$\therefore\ 2^{x+1} = 3^{x+1}$

$\therefore\ x + 1 = 0$

$x = -1$

2 $4^{x^2-9} = 1$

$\therefore\ x^2 - 9 = 0$

$\therefore\ x^2 = 9$

$\therefore\ x = \pm\sqrt{9}$

$x = \pm 3$

3 $4^{x^2-9} = 1$

$\therefore\ x^2 - 9 = 0$

$\therefore\ x^2 = 9$

$\therefore\ x = \pm\sqrt{9}$

$x = \pm 3$

Example 4

Find n in each of the following :

1 $\frac{12^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ \frac{12^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ \frac{(3 \times 2^2)^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ \frac{3^{n-1} \times 2^{2(n-1)}}{2^{n-1} \times 3^{n-1}} = 1$

$\therefore\ 2^{2n-2-n+1} = 1$

$\therefore\ 2^{n-1} = 1$

$\therefore\ n - 1 = 0$

$\therefore\ n = 1$

2 $\frac{2^n \times 9^{n+1}}{18^n} = 3^n$

$\therefore\ \frac{2^n \times 9^{n+1}}{18^n} = 3^n$

$\therefore\ \frac{2^n \times (3^2)^{n+1}}{(2 \times 3^2)^n} = 3^n$

$\therefore\ \frac{2^n \times 3^{2(n+1)}}{2^n \times 3^{2n}} = 3^n$

$\therefore\ 3^{2n+2-2n} = 3^n$

$\therefore\ 3^2 = 3^n$

$\therefore\ n = 2$

Note 3

Solve both exponential equations in $\mathbb{R}$.

$x^2 \times 2^x = 10$

$\left(\frac{1}{2}\right)^{2x-1} = 5^{1-2x}$

Note 4

Find n in each of the following :

$\frac{x^n \times 9^{1-x}}{6^{2x}} = 1$

$\frac{2^x \times 3^x}{12^x} = \frac{1}{2}$

Key Point

- When the bases are the same, equate the exponents.
- When the bases are different, try to simplify and rewrite using the same base.
- **Remember :** $a^0 = 1$ (where $a \neq 0$).
- $a^n = b^n$ with $a, b > 0$ and $a \neq b$ implies $n = 0$.

Math Tip

Always simplify expressions before equating the exponents. Factorization and laws of exponents make the work easier.

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